Extended Kindergarten Rule and Sustainable Development *

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Abstract

This study constructs an endogenous growth model that incorporates environmental problems. Stokey (1998) and Aghion and Howitt (1998, Ch. 5) construct growth models with environmental externalities. In both models, the growth rate becomes positive and the environment improves over time only when the intertemporal substitution of consumption is not so elastic.

Brock and Taylor (2004) construct another type of growth model that incorporates the environment. They find that even if the intertemporal substitution of consumption is elastic, sustainable growth can be attained if optimal environmental policies are enforced. We extend the model introduced by Brock and Taylor (2004) to consider the role of technical progress and R & D for sustainable growth. First, we show that the productivity of the research sector is important for sustainable growth. Second, we show that the value of the intertemporal substitution of consumption is neither a necessary nor sufficient condition for sustainable growth. Third, results indicate that government can attain a socially optimum outcome if appropriate policies (R & D policy and environmental policy) are enforced.

Keywords: kindergarten rule, innovation, environmental externalities

1 Introduction

In this paper, we construct a simple endogenous growth model to examine problems that are related to sustainable growth. Several efforts have been made with regard to such problems: Stokey (1998), Gradus and Smulders (1993), Aghion and Howitt (1998, Ch. 5), Brock and Taylor (2004), and others.

Stokey (1998) constructs an AK model with environmental externalities. She finds that a necessary condition for sustainable growth is that the intertemporal substitution of

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consumption is not so elastic. Aghion and Howitt (1998) extend her model to include the role of technical progress and R & D for sustainable growth. They show that the main conditions for sustainable growth are little changed, even if the research sector is introduced. That is, the growth rate becomes positive and the environment improves over time only when the intertemporal substitution of consumption is not so elastic.

Brock and Taylor (2004) develop an alternative growth model with environmental pollution. One important difference between Brock and Taylor (2004) and the models introduced by Stokey (1998) and Aghion and Howitt (1998) is the role of intertemporal substitution of consumption. In the model of Brock and Taylor (2004), the intertemporal substitution of consumption does not play an important role for sustainable growth. Even when intertemporal substitution of consumption is elastic, sustainable growth can be attained.

The model introduced by Brock and Taylor (2004) ignores the importance of research activities. Therefore, we extend their model to consider the relationship between R & D-based economic growth and the environment. It can be said that our model incorporates the work of Brock and Taylor (2004) and characteristics of an R & D based endogenous growth model like those of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). One of our contributions is to clarify the process of technical progress (because of which the growth becomes sustainable) by introducing the role of research activities, if we compare our model with Brock and Taylor (2004). As in Brock and Taylor (2004), the value of the intertemporal substitution of consumption does not play an important role for sustainable growth. For that reason, if we compare our model with typical endogenous growth models that incorporate environmental problems (e.g., Stokey (1998), Aghion and Howitt (1998. Ch. 5)), our contribution is to present an alternative R & D-based endogenous growth model that includes environmental parameters.

This paper is organized as follows. Section 2 introduces the endogenous growth model with pollution. In section 3, we consider the social planner's problem. In section 4, we specifically examine the Balanced Growth Path (BGP) and show the conditions for sustainable growth. In section 5, the market economy is analyzed. In section 6, BGP in the market economy and optimal policy are derived. Lastly, in section 7, the main conclusions of this paper are described.

2 The Model

2.1 Final Goods and Pollution

This section describes the model that is considered in this paper. We begin by considering the final good sector. The final good is a homogeneous good. The production function is specified as

$$Y^G = AK^{\alpha}\left[\int_0^n x_i^{\xi} di\right]^{\frac{1-\alpha}{\xi}}, \tag{2.1}$$

where Y^G is the gross output[1], A is the productivity parameter, K is the capital stock, n denotes a measure (number)[2] of the available intermediate goods, $x_i (i \in [0, n])$ is the quantity of ith intermediate good used for production activities, $\alpha (0<\alpha<1)$ and $\xi (0<\xi<1)$ are the parameters.

We assume that some portion of Y^G becomes abatement input, which is denoted by Y^A. Following Brock and Taylor (2004), we call $Y \equiv Y^G - Y^A$ net output that can be consumed or invested.

Let us specify the pollution. We assume that pollution is caused only by the production process. For simplicity, pollution that originates in consumption or other activities is not examined. Following Brock and Taylor (2004), we assume that the flow of pollution is equal to pollution created minus the pollution abated. Gross output is positively correlated with pollution. Every unit of economic activity Y^G generates one unit of pollution as a joint product of output. In contrast, abatement activities (the input of such activities is denoted by Y^A) can reduce pollution. We assume that one unit of input Y^A can reduce $(a>1)$ units of pollution. Consequently, the flow of the pollution (denoted by D) is represented as

$$D = Y^G - aY^A \equiv Y^G[1 - a\theta], \tag{2.2}$$

where θ is defined as $\theta \equiv \frac{Y^A}{Y^G} \in [0, 1]$ (note that net output must not be smaller than zero). Furthermore, $1 - a\theta \geq 0$ must hold because the pollution flow cannot be negative (it is assumed that abatement can only reduce the pollution flow). The definition of θ and this condition imply $0 \leq \theta \leq \frac{1}{a} \equiv \theta^K$. We can regard θ as the intensity of abatement. From that assumption, we can say that

$$Y = AK^{\alpha}\left[\int_0^n x_i^{\xi} di\right]^{\frac{1-\alpha}{\xi}} (1-\theta). \tag{2.1}$$

Equations (2.2) and (2.3) imply that pollution increases as net output increases for given $Y^G \equiv AK^{\alpha}\left[\int_0^n x_i^{\xi} di\right]^{\frac{1-\alpha}{\xi}}$. Note that if we assume $\alpha = 1$, our model becomes the same as that of Brock and Taylor (2004).

2.2 R & D and Intermediate goods

Next we examine the research sector. Intermediate goods are differentiated horizontally and

1) More precisely, we should write $Y^G(t)$ instead of Y^G because it depends on time. We suppress (t) throughout this paper to simplify the notation.

2) We take the product space of the intermediate goods to be continuous rather than discrete and ignore integer constraints on the number of goods.

innovation is interpreted as expanding their numbers in this paper. Obtaining a blueprint of a new intermediate good requires resources (in our model, labor) devoted to the R & D sector. The production function of the R & D sector is defined as

$$\dot{n}=\varepsilon n L_R, \tag{2.4}$$

where the dot represents differentiation with respect to time such as $\dot{n}\equiv\frac{dn}{dt}$, ε is the productivity parameter and L_R is labor input in the research lab. The n on the right hand side denotes the spillover effects (positive externalities from the existing goods or technologies).

Each intermediate good is produced by a single input, labor. For any $i(i\in[0, n])$, one unit of labor is necessary to produce one unit of an intermediate good, which implies that the labor demand in this sector, L_X, is equal to $\int_0^n x_i di(\equiv X)$. Note that labor is used to invent the new blueprint or to produce the intermediate goods.

2.3 Preferences

Next, we shall deal with consumers. They have utility over an infinite horizon. Following Stokey (1998), we specify the objective function of the representative consumer as

$$U=\int_0^{\infty} e^{-\rho t}\left(\frac{c^{1-\sigma}-1}{1-\sigma}-BS^{\gamma}\right)dt, \tag{2.5}$$

where $\rho(>0)$ is the subjective discount rate, c represents the per-capita consumption, $B(>0)$ is the parameter showing how much each individual suffers from pollution stock, and S is that pollution stock. We also assume that $\sigma>0$ and $\gamma>0$. It is assumed that the pollution stock affects utility. Following Stokey (1998), Aghion and Howitt (1998, Ch. 5), and Brock and Taylor (2003), we assume that the dynamic behavior of pollution stock is specified as

$$\dot{S}=D-\eta S, \tag{2.6}$$

where $\eta(>0)$ denotes the rate at which the pollution stock decays.

3 Social Optimum and Kindergarten Rule

3.1 Social Planner's Problem

The social planner's problem is to maximize (2.5), subject to

$$\dot{K}=AK^{\alpha}\left[\int_0^n x_i^{\xi} di\right]^{\frac{1-\alpha}{\xi}}(1-\theta)-C, \tag{3.1}$$

$$\frac{1}{\varepsilon}\frac{\dot{n}}{n}+\int_0^n x_i di=L, \tag{3.2}$$

and (2.6), $K(0)=K_0$, $n(0)=n_0$, $S(0)=S_0$[3], where $C\equiv cL$ represents the total consumption and L (which is constant over time) represents the population in this economy.

The current value Hamiltonian for this problem takes the form of

$$\mathcal{H}=\frac{c^{1-\sigma}-1}{1-\sigma}-BS^{\gamma}+\mu_1(AK^{\alpha}n^{\frac{1-\alpha}{\xi}}x^{1-\alpha}(1-\theta)-cL)$$
$$+\mu_2(AK^{\alpha}n^{\frac{1-\alpha}{\xi}}x^{1-\alpha}(1-a\theta)-\eta S)+\mu_3(\varepsilon n(L-nx)), \tag{3.3}$$

where x is the quantity of each intermediate good, μ_1, μ_2, and μ_3 respectively denote the shadow prices of capital, pollution stock and the measure of the intermediate goods. Note that the conditions, $\frac{1}{\varepsilon}\frac{\dot{n}}{n}+\int_0^n x_i di=L$ and $x_i=x$ are already used. From the previous discussion, $L_X=\int_0^n x_i di$. However, according to the problem to maximize $\left[\int_0^n x_i^{\xi} di\right]^{\frac{1-\alpha}{\xi}}$, subject to the constraint $L_X=\int_0^n x_i di$, we can show that $x_i=x$ and $L_X=nx$. Therefore, $\dot{n}$ is equal to $\varepsilon n(L-nx)$. The parameter μ_2 becomes negative because it is the shadow price of the pollution stock.

Observe the term involving the control variable θ,

$$Max\ \ Y^G\theta(-\mu_1-a\mu_2),\ s.t.\ 0\leq\theta\leq\theta^K\equiv\frac{1}{a}.$$

The optimal value of θ depends on the value of the term $Z\equiv-\mu_1-a\mu_2$. When $Z<0$, $\theta=0$ because the shadow value of capital is high relative to that of pollution. In this case, the economy is poor and capital accumulation is more important than abatement. In other words, abatement is expensive and zero abatement will occur. When $Z>0$, $\theta=\theta^K$ because the shadow value of capital is low relative to that of pollution. In this case, the economy is rich. Abatement is therefore cheap and maximal abatement will be undertaken. If $Z=0$ we can say that abatement might occur, but it is not necessarily maximal.

When $Z<0$, the conditions for the maximum can be written as

$$\left.\begin{aligned}
c^{-\sigma}&=\mu_1 L,\\
\theta&=0,\\
X&=\frac{1-\alpha}{\varepsilon\mu_3 n}(\mu_1+\mu_2)Y^G,\\
-\frac{\dot{\mu}_1}{\mu_1}&=\alpha\left(\frac{Y^G}{\mu_1 K}\right)(\mu_1+\mu_2)-\rho,\\
-\frac{\dot{\mu}_2}{\mu_2}&=-\rho-\eta-\frac{B\gamma S^{\gamma-1}}{\mu_2},\\
\text{and}\quad -\frac{\dot{\mu}_3}{\mu_3}&=-\rho+\frac{1-\alpha}{\xi n\mu_3}(\mu_1+\mu_2)Y^G+\varepsilon(L-2X).
\end{aligned}\right\} \tag{3.4}$$

3) The (0) notation denotes the level of time 0. For example, $K(0)$ is the level of physical capital at time 0. Note that the initial value of each stock variable is given.

When $Z=0$, we have an interior solution for θ and the conditions for the maximum can be written as

$$\left.\begin{aligned} c^{-\sigma}&=\mu_1 L,\\ \theta&\in[0,\theta^K],\\ X&=\frac{1-\alpha}{\varepsilon\mu_3 n}(\mu_1 Y^G(1-\theta^K)),\\ -\frac{\dot{\mu}_1}{\mu_1}&=\alpha\left(\frac{Y^G}{K}\right)(1-\theta^K)-\rho,\\ -\frac{\dot{\mu}_2}{\mu_2}&=-\rho-\eta-\frac{B\gamma S^{\gamma-1}}{\mu_2},\\ \text{and}\quad -\frac{\dot{\mu}_3}{\mu_3}&=-\rho+\frac{1-\alpha}{\xi n\mu_3}(\mu_1 Y^G(1-\theta^K))+\varepsilon(L-2X). \end{aligned}\right\} \tag{3.5}$$

When $Z>0$, maximal abatement will occur and the conditions for the maximum can be written as follows:

$$\left.\begin{aligned} c^{-\sigma}&=\mu_1 L,\\ \theta&=\theta^K,\\ X&=\frac{1-\alpha}{\varepsilon\mu_3 n}\mu_1 Y^G(1-\theta^K),\\ -\frac{\dot{\mu}_1}{\mu_1}&=\alpha\left(\frac{Y^G}{K}\right)(1-\theta^K)-\rho,\\ -\frac{\dot{\mu}_2}{\mu_2}&=-\rho-\eta-\frac{B\gamma S^{\gamma-1}}{\mu_2},\\ \text{and}\quad -\frac{\dot{\mu}_3}{\mu_3}&=-\rho+\frac{1-\alpha}{\xi n\mu_3}\mu_1 Y^G(1-\theta^K)+\varepsilon(L-2X). \end{aligned}\right\} \tag{3.6}$$

Because $\theta=\theta^K$, the net emission of pollution becomes zero[4). Following Brock and Taylor (2004), this situation is called the "Kindergarten rule." Brock and Taylor (2004) use this phrase because pollution is cleaned up at the moment it is created when $\theta=\theta^K$. They describe

4) In all cases, the transversality conditions are given as

$$\lim_{t\to\infty} e^{-\rho t}\mu_1 K=0,$$
$$\lim_{t\to\infty} e^{-\rho t}\mu_2 S=0,$$

and

$$\lim_{t\to\infty} e^{-\rho t}\mu_3 n=0.$$

that (by quoting the book by Fulghum (1986)) "This is one of the most common rules taught in kindergarten".

4 BGP and Long-Run Growth

4.1 BGP when $Z>0$

Consider the growth path with active and maximal abatement. In the BGP, each variable grows at a constant rate, which in turn implies Y, K and C grow at the same rate. This rate is given as

$$g_Y^* = \frac{1}{\sigma} \left(\frac{1-\xi}{\xi} \varepsilon L - \rho \right), \tag{4.1}$$

where g_j denotes the growth rate of variable j. We use the asterisk here to denote the socially optimal level.[5] We assume that $\frac{1-\xi}{\xi} \varepsilon L - \rho > 0$, so that $g_Y^* > 0$.

How about the dynamic behavior of the pollution? Note that the pollution flow must be zero because $\theta = \theta^K$. In this situation,

- Y, K and C grow exponentially over time with the rate given as (4.1).
- θ is set to the Kindergarten rule level (that is, $\theta = \theta^K$).
- The following equation must be satisfied:

$$\frac{\dot{S}}{S} = -\eta. \tag{4.2}$$

The environment improves at the rate of η in the long run and the economy approaches a pristine level of environmental quality. Note that σ does not play an important role for $\dot{S}<0$. Unlike the model of Stokey (1998) and Aghion and Howitt (1998, Ch. 5), even when the intertemporal substitution of consumption is elastic, that is, even if $\sigma<1$, sustainable growth can be attained. Stokey (1998) and Aghion and Howitt (1998, Ch. 5) also construct growth models with environment. In their model $\sigma>1$ is a critical condition for sustainable growth.

Brock and Taylor (2004) present three stylized facts about growth and environment. One is that in most OECD countries, expenditures on pollution abatement costs per GDP show a somewhat upward, but roughly constant trend. They show that in the United States, business expenditures on pollution abatement costs per GDP after 1980 are constant. During 1975-1990, the United States devoted about 1.6-1.8% of GDP to pollution abatement activities. We can obtain similar results from observation of data of the other OECD countries. Total expenditures in France rose from 1.2% of GDP in 1990 to 1.6% in 2000. In Germany, the costs rose from 1.4% of GDP in 1990 to 1.6% in 2000. Austria and Netherlands show somewhat higher expenditures. Austria and the Netherlands devote 2.1% and 1.6% of GDP

5) For derivation of (4.1), see the Appendix.

to pollution abatement activities in 1990; those expenditures rose to 2.6% and 2.0% in 1998. These descriptions show that pollution abatement costs in OECD countries are about 1-2% of GDP and are apparently roughly constant (although they show a somewhat upward trend).[6)]

In our model, θ can be regarded as representing expenditures on pollution abatement costs per GDP. Therefore, we might say that θ^K is about 0.02.

4.2 Long-Run Equilibrium when $Z=0$

Next, we consider the case where $Z=0$. In this case $\frac{\dot{\mu}_1}{\mu_1}=\frac{\dot{\mu}_2}{\mu_2}$ because $\mu_1=-a\mu_2$. From (3.5), we can obtain

$$\alpha\frac{Y^G}{K}(1-\theta^K)+\eta=-\frac{B\gamma S^{\gamma-1}}{\mu_2}.$$

It can be modified to

$$\alpha(1-\theta^K)\frac{Y^G}{K}+\eta=-\frac{aB\gamma S^{\gamma-1}}{\mu_1}. \tag{4.3}$$

$\frac{S^{\gamma-1}}{\mu_1}$ must be constant in the long run because the right hand side of (4.3) is constant in the long run. It means that

$$(\gamma-1)g_S=\frac{\dot{\mu}_1}{\mu_1}=-\sigma g_Y \tag{4.4}$$

in the long run.

Equation (2.6) can be rewritten as

$$\frac{\dot{S}}{S}=\frac{Y^G(1-a\theta)}{S}-\eta. \tag{4.5}$$

From (4.5), we can get

$$1-a\theta=(g_S+\eta)\frac{S}{Y^G}>0$$

if $1-a\theta>0$. From (4.4), $g_S=\frac{-\sigma}{\gamma-1}g_Y$. Therefore, $g_S+\eta>0$ implies

$$\sigma g_Y^*<\eta(\gamma-1).$$

In the long run, the values of θ approach the Kindergarten rule level, θ^K. The growth rate and dynamic behavior of pollution stock approach

6) See Brock and Taylor (2004) for details of the discussion.

$$g_Y^* = \frac{1}{\sigma}\left(\frac{1-\xi}{\xi}\varepsilon L - \rho\right),$$

and

$$\frac{\dot{S}}{S} = \frac{-\sigma}{\gamma-1} g_Y^*,$$

respectively.

4.3 BGP when $Z<0$

If $Z<0$, $\theta=0$; therefore, $g_S=g_Y$. This situation contradicts sustainable growth. For that reason, the remainder of the paper specifically addresses the case in which $Z>0$ and $Z=0$.

4.4 Kindergarten Rule and the Interior Solution

In 4.1 and 4.2, we derived two types of BGPs. One is $Z>0$ and maximal abatement occurs (kindergarten rule is satisfied); another is $Z=0$ and abatement is active but not maximal.

First, we will show that if $\sigma g_Y^* < \eta(\gamma-1)$, then $\theta \neq \theta^K$ along the BGP. Suppose that $Z>0$ ($-\mu_1 > a\mu_2$) and $\theta=\theta^K$. Then, $\mu_1=\mu_1(0)e^{g_{\mu_1}t}=\mu_1(0)e^{-\sigma g_Y t}$ and $\mu_2=\mu_2(0)e^{g_{\mu_2}t}=\mu_2(0)e^{-\eta(\gamma-1)t}$ along the BGP. When $-\mu_1>a\mu_2$,

$$\frac{\mu_1(0)}{a\mu_2(0)} < e^{[-\eta(\gamma-1)+\sigma g_Y]t}$$

must be satisfied. However, the right hand side approaches 0 as t goes to infinity because $-\eta(\gamma-1)+\sigma g_Y<0$ by assumption. It is a contradiction.

We can also show that if $\sigma g_Y^* > \eta(\gamma-1)$, then $\theta=\theta^K$ along the BGP.

5 The Market Economy

In this section, a decentralized economy shall be considered. First of all, we shall analyze the final good sector. The market for the final good is assumed to be perfectly competitive. Many firms manufacture homogeneous final goods subject to the same technology given as (2.1).

Firms maximize their profits at each date, taking the interest rate, r, the number of intermediate goods, n, the prices of intermediate goods, $p_i (i \in [0, n])$, and tax rate, τ, as given. We assume that a government can impose a tax on the final good. Tax revenue becomes abatement input. That is, $\tau Y^G = Y^A$. We also assume that the price of the final good is normalized to 1. A profit function Π is given as

$$\Pi = (1-\tau) A K^{\alpha} \left[\int_0^n x_i^{\xi} di\right]^{\frac{1-\alpha}{\xi}} - rK - \int_0^n p_i x_i di, \tag{5.1}$$

From the firms' profit maximization we can obtain

$$r=(1-\tau)\alpha\frac{Y^{G}}{K}, \tag{5.2}$$

$$x_i=\frac{\int_0^n p_i x_i di}{\int_0^n p_i^{-\frac{\xi}{1-\xi}} di} p_i^{-\frac{\xi}{1-\xi}}. \tag{5.3}$$

In equilibrium, the profit of the final goods sector must be zero, which means that

$$(1-\tau)(1-\alpha)Y^{G}=\int_0^n p_i x_i di$$

must hold in equilibrium.

Firms might enter freely into the R & D sector. They finance that cost by issuing equity and employing workers to obtain blueprints for new intermediate goods. They become able to produce monotonically over time if they invent successfully. Therefore, we assume that the inventor of an intermediate good of line j retains a perpetual monopoly right over the production and sale of jth intermediate good. Again, the production function in this sector is given as (2.4).

In the intermediate goods sector, firms produce goods using the blueprints that they created in the R & D sector. The profit function of firm i is given as

$$\pi_i=p_i x_i-w x_i \tag{5.4}$$

where w is the wage rate and π_i is the profit of firm i. Because the demand function for firm i is (5.3), it maximizes the profit by setting p_i as

$$p_i=p=\frac{w}{\xi} \tag{5.5}$$

Equation (5.5) shows that the price of each intermediate good and the profit that each firm can earn are equal in every industry at any given moment in time. We will denote $x_i=x$ and $\pi_i(\in[0,\ n])$ as π.

Next, the value of each R & D project is considered. A firm that succeeds in research activities at a particular time can subsequently earn profits by supplying the intermediate good monotonically to the final goods sector. Consequently, the value of each R & D project can be expressed as

$$v=\int_t^{\infty} e^{-\int_t^{t'} r(\eta)d\eta}\pi(t')dt', \tag{5.6}$$

where v is the value of the equity of each firm. From (5.6), we can obtain the no-arbitrage condition:

$$rv=\pi+\dot{v}. \tag{5.7}$$

Note that the right hand side of (5.7) is the total return to the owners of each firm. On the other hand, the left-hand side is the return to investors in the form of no-risk loans. Because of the equilibrium in the capital market, these values must be equal.

From (2.4), the labor needed to develop one unit of the intermediate good is $\frac{1}{n\varepsilon}$. The value created by such activity is v. Therefore, according to the assumption of free-entry to R & D, we obtain

$$v \leq \frac{w}{n\varepsilon}. \tag{5.8}$$

If $\dot{n}>0$, then (5.8) must hold with equality. If $v>\frac{w}{n\varepsilon}$, firms must employ as much labor as possible. Nevertheless, this situation never occurs in the state of equilibrium. On the contrary, if $v<\frac{w}{n\varepsilon}$, firms must choose $\dot{n}=0$ for their optimization.

Let us next describe consumers. Individuals earn wages by supplying labor, the interest from their assets. They decide how much they will consume and save in order to maximize their utilities over an infinite horizon given as (2.5), They take the path of r, w, S as given. They maximize (2.5) subject to their budget constraints

$$\dot{b}=rb+w-c, \tag{5.9}$$

where b denotes per-capita assets and $b(0)$ (the initial value of the assets) is given as b_0.

The conditions for the maximum are

$$c^{-\sigma}=\mu_4, \tag{5.10}$$

$$\dot{\mu}_4-\rho\mu_4=-r\mu_4, \tag{5.11}$$

and the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\mu_4 a=0, \tag{5.12}$$

where μ_4 is the shadow price of the income. From (5.10) and (5.11), the growth rate of consumption is given as

$$g_c=\frac{1}{\sigma}(r-\rho). \tag{5.13}$$

6 Steady State and Optimal Policy

The steady state is again addressed in the discussion of this section. We assume that τ is constant (or approaches a constant level) along the BGP. Consequently, the relationship between g_Y and g_n is represented as before.

Two equations (the labor market clearing condition and the no-arbitrage condition) are used to derive the growth rate of each variable. The labor market clearing condition is

$$\frac{1}{\varepsilon}\frac{\dot{n}}{n}+X=L.$$

From (5.8), (5.13), and $\frac{\dot{v}}{v}=g_Y-g_n$, the no-arbitrage condition (5.7) can be rewritten as

$$\frac{1-\xi}{\xi}\varepsilon X=g_n+(\sigma-1)g_Y+\rho. \tag{6.1}$$

Therefore, in a market economy, the growth rate, denoted as g_Y^d becomes the following:

$$g_Y^d=\left(\sigma+\frac{\xi}{1-\xi}\right)^{-1}\left(\frac{\xi}{1-\xi}\varepsilon L-\rho\right). \tag{6.2}$$

In this paper, superscript d indicates the level of the steady state in the market economy. Note that g_Y^d differs from g_Y^* because externalities are associated with the R & D sector in a market economy. In (6.2), the variables in the first parentheses are

$$\left(\sigma+\frac{\xi}{1-\xi}\right)^{-1}$$

rather than σ^{-1}, thereby reflecting the externalities related to spillover effects. As noted previously, each research activity contributes to the stock of general knowledge capital that will be useful to later generations of researchers. This spillover effect tends to make g_Y^d too low. For that reason, resources devoted to the research sector tend to be too small in a market economy because of this effect.

For those reasons, a policy that corrects the market distortions must be considered. Suppose that the government pays a fraction ϕ of the research cost. If such a policy is carried out, then the free-entry conditions become

$$\varepsilon n v=w(1-\phi). \tag{6.3}$$

Consequently, the no-arbitrage conditions become

$$\frac{\frac{1-\xi}{\xi}\varepsilon X}{1-\phi}=g_n+(\sigma-1)g_Y+\rho. \tag{6.4}$$

The labor-market clearing condition and the relationship between g_n and g_Y are as before. According to a simple calculation, an optimal subsidy rate (which is denoted by ϕ^*) to achieve g_Y^* is shown as

$$\phi^*=\frac{g_n^*}{g_n^*+(\sigma-1)g_Y^*+\rho}. \tag{6.5}$$

Resources devoted to the research sector are too small in a market economy. Therefore, the government must subsidize R & D activities to make g_Y^d to g_Y^*. If the R & D policy described above is carried out appropriately, then various variables will grow at the socially optimum rate.

As described in the previous section, the government exercises power to protect the environment by taking taxes from firms that are producing final goods. However, this power is insufficient to achieve a social optimum because two market failures exist in this economy. The government should use (at least) two measures (environmental policy and R & D policy) to attain the social optimum.

Tax rate τ must be the optimal value of θ derived in section 3 and section 4. That is, τ must be θ^K if $\sigma g_Y^* > \eta(\gamma - 1)$ along the BGP. If $\sigma g_Y^* < \eta(\gamma - 1)$, $\tau \neq \theta^K$ but τ approaches to θ^K along the BGP. If we assume $\theta^K = 0.02$ as we discussed before, then the optimal tax rate will be 2% in the long run.

7 Concluding Remarks

This paper has presented an investigation of the relationship between economic growth and the environment. The model introduced by Brock and Taylor (2004) ignored the importance of research activities. Therefore, we extend their model to consider the relationship between R & D based economic growth and the environment. Our model incorporates the work of Brock and Taylor (2004) and R & D based endogenous growth model like Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

First, we showed that if the model is extended to the endogenous growth model with innovation and environment, many results are almost identical. The kindergarten rule is satisfied if the steady growth rate is high and the natural regeneration rate is low, and if people are less sensitive to the pollution stock. However, in our model, the growth rate depends on parameters related to R & D sector such as research productivity or substitutability of goods invented in the research sector. This dependence is not pointed out by Brock and Taylor (2004) because R & D is not considered in their model. That difference is the most important one between our model and that of Brock and Taylor (2004). In any case, the long-run growth rate becomes positive and the environment improves over time.

One contribution is clarification of the process of technical progress (because of which growth becomes sustainable) by introducing the role of research activities if we compare our model with Brock and Taylor (2004).

Then the market economy is considered. Two market failures exist in the market economy. First, no innovator internalizes a contribution to knowledge capital. Because this causes market failure, the growth rate in a market economy tends to be lower than that in the centralized economy. Second, nobody will engage in abatement activities because firms have no incentive to do so. We have derived an optimal R & D policy and an environmental policy that induce the market economy to grow at the socially optimal rate. The growth rate in a market economy is too low. Therefore, a policy that promotes research activities is necessary. We also show that government must impose a tax on the final good (this policy can be regarded as environmental policy because output is positively correlated with pollution) to control the pollution level appropriately.

As in Brock and Taylor (2004), the value of σ does not play an important rule in sustainable growth. Note that famous works such as that by Stokey (1998), and Aghion and Howitt (1998, Ch. 5) conclude that $\sigma > 1$ is an essential condition for sustainable growth. However, this result is no longer true in our model. Therefore, if we compare our model with typical growth models with an environment such as that posited by Stokey (1998) and Aghion and Howitt (1998, Ch. 5)), our contribution is to present an alternative R & D-based endogenous growth model with environment.

The results described here suggest several areas for further research. One simple idea is that innovation is interpreted to improve the quality of the goods rather than to expand the number of goods. It might also be interesting to extend this model to a framework of the international economy, especially a North-South model. Using such a framework, we can consider the effects of trade or pollution that spills across national boundaries. Another interesting idea is the introduction of a Clean Development Mechanism by which a developed country and a less developed country can cooperate to reduce pollution.

In addition, a focus on the relationship between growth and the environment should introduce innovation that is intended to reduce pollution. The problem of whether or not entrepreneurs in the economy have sufficient incentive to engage in environmental R & D must be considered.

A Appendix

A. 1 Growth Rate Derivation

Here, the growth rates of various variables are derived. Because Y, K and C grow at the same rate in the steady state, (2.8), (2.9) and (2.10) imply that

$$g_Y = \frac{1}{\sigma} g_{\mu_1}, \tag{A.1}$$

and

$$g_Y + g_{\mu_1} = g_n + g_{\mu_3} = (1-\sigma) g_Y. \tag{A.2}$$

Because g_{μ_2} must be constant along the BGP, we can obtain

$$(\gamma-1) g_S = g_{\mu_2} = -\eta(\gamma-1). \tag{A.3}$$

From (2.4), (A.2), we can show that

$$(\sigma-1) g_Y = \frac{1-\xi}{\xi}(\varepsilon X - \rho). \tag{A.4}$$

The production function (2.3) can be rewritten as

$$g_Y = \frac{(1-\xi)}{\xi} g_n. \tag{A.5}$$

From (A.4) and (A.5), the long-run growth rate becomes

$$g_Y^* = \frac{1}{\sigma}\left(\frac{1-\xi}{\xi}\varepsilon L - \rho\right).$$

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