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# Analysts Reports, Stock Prices, and Reputation Concern

Manabu Kotani

熊　本　学　園　大　学
産　業　経　営　研　究　所

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Manabu Kotani

Kumamoto Gakuen University

**ABSTRACT**

This study demonstrates that stock price considerations can generate an optimism bias in analysts reports. Based on a microstructure framework, we derive an equilibrium consisting of the analyst's reporting strategy and the informed trader's trading strategy. We reveal that an equilibrium exists in which analysts adopt an optimistic reporting strategy to stimulate buy orders, and informed traders choose a trading strategy that follows analyst reports. Furthermore, there are two equilibria, one in which relatively honest reports are provided and one in which dishonest reports are frequently issued. The results also reveal how analyst honesty affects the optimism of reports, the news value to informed traders, and the probability distribution of stock prices. The economic significance of analyst reports differs depending on which of the two equilibria is realized.

***Keywords***— microstructure, informed trader, market maker, optimism, perfect Bayesian equilibrium

## 1. Introduction

It is widely known that earnings forecasts and stock recommendations of sell-side analysts contain an optimistic bias[1]. Analysts' motives for generating such bias include contributing to their investment banks (Dugar and Nathan, 1995; Lin and McNichols, 1998), building good relationships with the managers they cover (Francis and Philbrick, 1993; Lim, 2001), and earning stock commissions (Hayes, 1998; Jackson, 2005; Cowen et al., 2006).

In contrast, we demonstrate that stock price considerations can be a source of optimism bias[2]. We construct a model based on Jackson (2005) and the microstructure framework. Specifically, in our model, an analyst sends report to an informed trader, who observes the report and submits orders to a market maker. The market maker sets the stock price by observing net order flow from the informed trader and the noise trader. Under this setup, we obtain an equilibrium consisting of the analyst's reporting strategy and the informed trader's trading strategy.

Thus, Jackson (2005) is the previous study most closely related to ours. However, they differ in the following two respects. First, Jackson (2005) considers trading commissions and short-selling constraints as drivers that affect the characteristics of analyst reports,

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[1] See the survey by Ramnath et al. (2008), for example.

[2] Note that Morgan and Stocken (2003) focus on analysts who consider stock prices.

whereas we assume that stock price considerations are the drivers. The other is the assumption about the stock price. Specifically, Jackson (2005) assumes that investors are sufficiently small such that their trading has no effect on stock prices[3]. However, institutional investors who receive analyst reports are not minuscule players, and their trades will affect stock prices. In addition, it is reasonable to assume that in a rational stock market, analysts' motives are reflected in stock prices[4]. Therefore, we argue that allowing for stock price changes would yield more interesting implications.

Analysis determined that an equilibrium exists in which analysts adopt an optimistic reporting strategy to stimulate buy orders and informed traders adopt a trading strategy that follows analysts' reports. We further established that there are two equilibria, one in which relatively honest reports are issued and one in which more dishonest reports are issued. The results also reveal the effect of analyst honesty on the optimism of reports, the news value to informed traders, and the probability distribution of stock prices. Depending on which of the two types of equilibria is realized, the economic significance of analyst reports differs considerably. The major difference between our results and those of previous research is that we demonstrate that two types of equilibria exist.

The rest of this paper is organized as follows. The model setup is described in section 2. The equilibrium is derived in section 3. The main results with numerical examples are discussed in section 4. Finally, concluding remarkes are presented in section 5.

## 2. Model

Our model consists of the following players: a sell-side analyst, an informed trader, a market maker, and a noise trader. All players are risk neutral. Assume that the value per unit of a company's stock $x$ is known to be $x_H$ with probability 0.5 and $x_L$ with probability 0.5 (i.e., $x \in \{x_H, x_L\}$). Let $x_H > 0$ and $x_L = 0$. The analyst is divided into two types: the capable type with probability $\theta$ and the incapable type with probability $1-\theta$ $(0<\theta<1)$. The capable analyst receives useful private signal about the value of the firm. On the other hand, the private signal that the incapable analyst receives is noise. The analyst does not know whether she is a capable or incapable type. The private signal is either $s_H$ or $s_L$, meaning the firm value will be $x_H$ or $x_L$, respectively. The conditional probability of the private signal is expressed as follows:

$$Pr(s_H|x_H, capable) = q, \tag{1}$$

$$Pr(s_H|x_L, capable) = 1-q, \tag{2}$$

and

$$Pr(s_H|x_H, incapable) = Pr(s_H|x_L, capable) = 0.5. \tag{3}$$

We assume that $0.5 < q < 1$.

Upon receiving a private signal, the analyst sends a report $m$ to the informed trader. The report is either $m_H$ or $m_L$ (i.e., $m \in m_H, m_L$), meaning the firm value is $x_H$ or $x_L$, respectively. The private signals and the reports need not coincide. For example, if the private signal is $s_L$, it is permissible to send a report $m_H$. Upon receiving the report, the informed trader assesses the posterior probability $\lambda$ that $x_H$ has occurred.

Specifically, let us express the following

$$\lambda(m_H) \stackrel{\text{def}}{=} Pr(x_H|m_H) \tag{4}$$

and

$$\lambda(m_L) \stackrel{\text{def}}{=} Pr(x_H|m_L). \tag{5}$$

[3] Jackson (2005) states that these assumptions are made for the sake of analytical simplicity.
[4] For example, Lin and McNichols (1998) argue that some of the motives behind analyst reports are reflected in stock price.

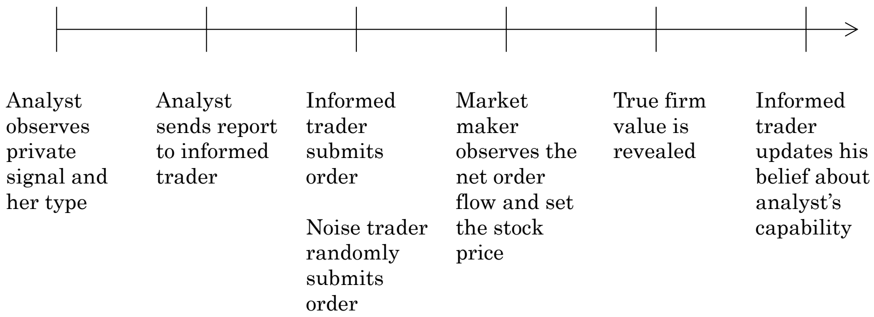


Figure 1: Timeline

From this, we can express $Pr(x_L|m_H) = 1 - \lambda(m_H)$ and $Pr(x_L|m_L) = 1 - \lambda(m_L)$.

To account for the rationality of stock prices, we introduce a microstructure into the model based on Krishnan (1992), who transforms Kyle' (1985) model into a binary type. First, suppose that the informed trader submits either a buy or a sell order in 1 unit. In the following, we denote the buy order of the informed trader as $a = 1$ and the sell order as $a = -1$. On the other hand, the noise trader submits an order randomly. Specifically, the noise trader submits a buy order in 1 unit with probability 0.5 and a sell order in 1 unit with probability 0.5. The noise trader's buy order is denoted by $u = 1$ and its sell order by $u = -1$. That is, $Pr(u = 1) = Pr(u = -1) = 0.5$. Market makers cannot identify order separately, but can observe only the net order flow $\omega (= a + u)$[5]. Suppose that market makers are exposed to competition and determine the stock price $P_\omega$ to absorb the net order flow. Since there are three possible values of $\omega$: 2, 0, and −2, we write $P_{\omega=2}$, $P_{\omega=0}$, and $P_{\omega=-2}$ for the prices set by the market maker depending on the value of $\omega$.

At the end of the game, the value of the firm is revealed. Let the informed trader update his beliefs about the analyst's capability (i.e., capable or incapable) based on the realized firm value $x$ (i.e., $x_H$ or $x_L$) and the analyst's report $m$ (i.e., $m_H$ or $m_L$). The updated informed trader's belief is expressed as:

$$\hat{\theta} \overset{\text{def}}{=} Pr(capable \mid x, m). \tag{6}$$

We call $\hat{\theta}$ the analyst's reputation. The timeline is illustrated in Figure 1.

Let the analyst's objective function be expressed as follows:

$$\max_{m \in \{m_H, m_L\}} E(P_\omega | m) + k \cdot E(\hat{\theta} | m), \tag{7}$$

where the first and second terms are the conditional expectation of the stock price $(P_\omega)$ and expectation of reputation $(\hat{\theta})$, respectively. Thus, we assume that the analyst pursues both stock price and reputation. The parameter $k$ represents how much the analyst values reputation: it is a random variable that follows a uniform distribution in the interval [0, 1]. Suppose that the analyst and the informed trader know the probability distribution of $k$ and further assume that the analyst can observe the realized value of $k$. We define the threshold $k^*$ of $k$ as follows. If $k \geq k^*$,

[5] For simplicity of analysis, the order volumes of informed traders and noise traders are exogenously determined in this study. However, the essence of the microstructure, in which private information is inferred by market makers observing net order flow, is not lost.

then the analyst sends report $m_H$ when she observes signal $s_H$ and report $m_L$ when she observes signal $s_L$. That is, the analyst honestly issues a report consistent with the signal. On the other hand, if $k < k^*$, the analyst not only sends $m_H$ when she observes the signal $s_H$, but also when she observes $s_L$. That is, the analysts issues $m_H$ no matter what signal she receives. In this study, if the analyst type $k$ is $k \geq k^*$, she is referred to as a good type, and if $k < k^*$, an evil type. We denote the probability that an analyst is good type from the informed trader's point of view by $\pi$. That is

$$\pi \stackrel{\text{def}}{=} Pr(k \geq k^*). \tag{8}$$

From this, the probability that an analyst is evil is expressed as $1 - \pi$.

Since the analyst knows whether she is good or evil, her information set can be classified into four categories according to the combination of the signal and her type: $\{s_H, good\}$, $\{s_H, evil\}$, $\{s_L, good\}$, and $\{s_L, evil\}$. The analyst's strategy is a mapping from these four information sets to a report $m_H$ or $m_L$. On the other hand, the informed trader's information set can be classified into $\{m_H\}$ and $\{m_L\}$ according to the reports. The strategy of the informed trader is to map these two information sets to a buy order (i.e., $a = 1$) or a sell order (i.e., $a = -1$).

## 3. Equilibrium

In this section, we derive a pure-strategy perfect Bayesian equilibrium. This equilibrium concept requires that, given their beliefs, the strategy profile of each agent is sequentially rational, and that, where possible, beliefs should be determined by Bayes' rule from the players' equilibrium strategies.

***Proposition.***

Let $k \geq k^*$, then the analyst is of the good type, and let $k < k^*$ be of the evil type. The threshold $k^*$ is expressed as follows:

$$k^* = \frac{\pi^2 \theta^2 (2q-1)^2 - (\pi - 2)^2}{4(2-\pi)\pi(\theta - 1)\theta(2q-1)} \cdot x_H. \tag{9}$$

The analyst sends $m_H$ at the information sets $\{s_H, good\}$, $\{s_H, evil\}$, and $\{s_L, evil\}$, and $m_L$ at $\{s_L, good\}$. The informed trader submits a buy order (i.e., $a = 1$) when he receives $m_H$ and a sell order (i.e., $a = -1$) when he receives $m_L$. The belief of the informed trader is $\lambda(m_H)$ when he receives $m_H$ and $\lambda(m_L)$ when he receives $m_L$. Here, we have

$$\lambda(m_H) = 0.5 \cdot \frac{2 - \pi + \theta\pi(2q-1)}{2-\pi},$$

and

$$\lambda(m_L) = 0.5 + \theta(0.5 - q).$$

***Proof.***

The proof consists of the following steps: First, given the analyst's strategy, we obtain the beliefs and best response of the informed trader. Next, under the assumption that both the analyst's strategy and informed trader's reaction are optimal, the stock price set by the market maker is obtained. Finally, given the optimal responses of informed traders and the stock price, we demonstrate that analysts have no incentive to change her strategy. In the following section, we provide a detailed proof.

**Best Response of the Informed Trader**

Given the analyst's strategy, we find the informed trader's best response when he receives $m_H$ and when he receives $m_L$.

- When the Informed Trader Receives the Report $m_H$.

Given the analyst's strategy, the posterior probability that the firm value is $x_H$ when $m_H$ is received (i.e., $\lambda(m_H)$) is calculated as follows

$$\lambda(m_H) = 0.5 \cdot \frac{2-\pi+\theta\pi(2q-1)}{2-\pi}. \qquad (10)$$

It is obvious that $\lambda(m_H) > 0.5$. That is, upon receiving $m_H$, the informed trader updates the probability that the firm value is $x_H$ upward from 0.5. Using $\lambda(m_H)$, the informed trader's expected profit $E\pi_{a=1,m_H}$ from a buy order (i.e., $a = 1$) is as follows:

$$\begin{aligned} E\pi_{a=1,m_H} = & 0.5[\lambda(m_H)(x_H - P_{\omega=2})) \\ & +\{1-\lambda(m_H)\}(x_L - P_{\omega=2})] \\ & +0.5[\lambda(m_H)(x_H - P_{\omega=0}) \\ & +\{1-\lambda(m_H)\}(x_L - P_{\omega=0})]. \end{aligned} \qquad (11)$$

On the other hand, the informed trader's expected profit $E\pi_{a=-1,m_H}$ from a sell order (i.e., $a = -1$) is as follows:

$$\begin{aligned} E\pi_{a=-1,m_H} = & 0.5\{-\lambda(m_H)(x_H - P_{\omega=0}) \\ & -(1-\lambda(m_H))(x_L - P_{\omega=0})\} \\ & +0.5\{-\lambda(m_H)(x_H - P_{\omega=-2}) \\ & -(1-\lambda(m_H))(x_L - P_{\omega=-2})\}. \end{aligned} \qquad (12)$$

From $\lambda(m_H)$ expressed in equation (10) and the stock price $P_\omega$ to be obtained later, $E\pi_{a=1,m_H} > E\pi_{a=-1,m_H}$ is satisfied[6]. In other words, given the analyst's strategy, the informed trader always submits a buy order when he receives $m_H$.

- When the Informed Trader Receives the Report $m_L$.

Given the analyst's strategy, the posterior probability that the firm value is $x_H$ when $m_L$ is received (i.e., $\lambda(m_L)$) is calculated as follows:

$$\lambda(m_L) = 0.5 + \theta(0.5 - q). \qquad (13)$$

From the above equation, $\lambda(m_L) < 0.5$. That is, upon receiving $m_L$, the informed trader updates the probability that the firm value is $x_H$ downward from 0.5. Using $\lambda(m_L)$, the informed trader's expected profit $E\pi_{a=1,m_L}$ from a buy order (i.e., $a = 1$) is as follows:

$$\begin{aligned} E\pi_{a=1,m_L} = & 0.5\{\lambda(m_L)(x_H - P_{\omega=2}) \\ & +(1-\lambda(m_L))(x_L - P_{\omega=2})\} \\ & +0.5\{\lambda(m_L)(x_H - P_{\omega=0}) \\ & +(1-\lambda(m_L))(x_L - P_{\omega=0})\}. \end{aligned} \qquad (14)$$

On the other hand, the informed trader's expected profit $E\pi_{a=-1,m_L}$ from a sell order (i.e., $a = -1$) is as follows:

$$\begin{aligned} E\pi_{a=-1,m_L} = & 0.5\{-\lambda(m_L)(x_H - P_{\omega=0}) \\ & -(1-\lambda(m_L))(x_L - P_{\omega=0})\} \\ & +0.5\{-\lambda(m_L)(x_H - P_{\omega=-2}) \\ & -(1-\lambda(m_L))(x_L - P_{\omega=-2})\}. \end{aligned} \qquad (15)$$

From $\lambda(m_L)$ expressed in equation (13) and the stock price $P(\omega)$ to be obtained later, we know that $E\pi_{a=-1,m_L} - E\pi_{a=1,m_L} > 0$ is satisfied[7]. This means that the informed trader always submits a sell order when he receives $m_L$.

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6 It can be shown that $E\pi_{a=1,m_H} - E\pi_{a=-1,m_H} = \frac{(1+\pi)\theta(q-0.5)}{2-\pi} x_H > 0$.

7 Note that $E\pi_{a=-1,m_L} - E\pi_{a=1,m_L} = 0.5\left\{\theta(q-0.5)\frac{3+4(1-\pi)+\theta(2q-1)}{2-\pi} + 2\pi\right\} x_H > 0$.

**Stock Price Set by Market Maker**

Given the analyst's strategy and the informed trader's response, the market maker infers the contents of the analyst's report from the net order flow and determines the stock price. In the following, we consider the cases when $\omega=2$, $\omega=-2$, and $\omega=0$.

- Stock Price in the Case of $\omega=2$

When $\omega=2$, the market maker infers that the analyst's report is $m_H$, both the informed trader and the noise trader submit a buy order. Therefore, the stock price $P_{\omega=2}$ is determined as follows:

$$P_{\omega=2}=\lambda(m_H)x_H+\{1-\lambda(m_H)\}x_L$$
$$=\left\{0.5+(q-0.5)\,\theta\frac{\pi}{2-\pi}\right\}x_H. \quad (16)$$

- Stock Price in the Case of $\omega=-2$

When $\omega=-2$, the market maker conjectures that the analyst's report is $m_L$, both the informed trader and noise trader submit a sell order. Therefore, the stock price $P_{\omega=-2}$ is determined as follows:

$$P_{\omega=-2}=\lambda(m_L)x_H+\{1-\lambda(m_L)\}x_L$$
$$=\{0.5-(q-0.5)\,\theta\}\,x_H. \quad (17)$$

- Stock Price in the Case of $\omega=-2$

When $\omega=0$, the market maker infers that one of the following two cases occurs: first, the analyst report is $m_H$, the informed trader submits a buy order, and the noise trader submits a sell order. In the other case, the analyst's report is $m_L$, the informed trader submits a sell order, and the noise trader submits a buy order. Therefore, the stock price $P_{\omega=0}$ is determined as follows:

$$P_{\omega=0}=Pr(m_H)\{\lambda(m_H)x_H+(1-\lambda(m_H))x_L\}$$
$$+Pr(m_L)\{\lambda(m_L)x_H+(1-\lambda(m_L))x_L\}. \quad (18)$$

Note that market makers, who cannot directly observe analyst reports, calculate the probability that $m_H$ and $m_L$ occur. Specifically, the probability that an analyst report is $m_L$ (i.e., $Pr(m_L)$) can be expressed as follows:

$$Pr(m_L)=0.5^2(1-\theta)\pi+0.5^2(1-\theta)\pi$$
$$+0.5\theta\pi+0.5\theta(1-q)\pi=0.5\pi. \quad (19)$$

Therefore, the following holds.

$$Pr(m_H)=1-0.5\pi. \quad (20)$$

Substituting $Pr(m_H)$ and $Pr(m_L)$ into equation (21) and rearranging, we obtain

$$P_{\omega=0}=0.5x_H. \quad (21)$$

**Analyst's Stock Price Considerations**

In the following, we identify the behavior of an analyst who considers the stock price. Using the stock prices $P_{\omega=2}$, $P_{\omega=0}$, and $P_{\omega=-2}$ obtained thus far, the expected stock price $E(P_\omega|m_H)$ is as follows:

$$E(P_\omega|m_H)=Pr(u=1)\cdot P_{\omega=2}+Pr(u=-1)\cdot P_{\omega=0}$$
$$=\left\{0.5+(q-0.5)\theta\frac{\pi}{4-2\pi}\right\}x_H. \quad (22)$$

On the other hand, the expected value of the stock price $E(P_\omega|m_L)$ is as follows:

$$E(P_\omega|m_L)=Pr(u=-1)\cdot P_{\omega=-2}+Pr(u=1)\cdot P_{\omega=0}$$
$$=\{0.5-0.5(q-0.5)\,\theta\}\,x_H. \quad (23)$$

Here, subtracting $E(P_\omega|m_H)$ from $E(P_\omega|m_L)$, we obtain

$$E(P_\omega|m_H)-E(P_\omega|m_L) = \left\{(q-0.5)\,\theta\frac{1}{2-\pi}\right\}x_H>0. \tag{24}$$

This indicates that $E(P_\omega|m_H)>E(P_\omega|m_L)$. In short, if the analyst considers the stock price, she issues $m_H$ regardless of whether the signal is $s_H$ or $s_L$.

**Analyst's Reputation Considerations**

In the following, we identify the behavior of an analyst who considers her reputation. When the analyst sends a report, she anticipates how informed traders assess her capabilities. At the time the analyst sends the report, it is unknown whether the firm value $x_H$ or $x_L$ is realized; therefore, the conditional expectation that the analyst forms is as follows:

- When $s_H$ is Received and $m_H$ is Issued

$$\begin{aligned}E(\hat{\theta}|m_H,s_H)&=Pr(x_H|s_H)\cdot Pr(capable|m_H,x_H)\\&\quad+Pr(x_L|s_H)\cdot Pr(capable|m_H,x_L)\\&=(0.5-0.5\theta+\theta q)\\&\quad\frac{(1-\pi+q\pi)\,\theta}{1-0.5\pi+(q-0.5)\,\pi\theta}\\&\quad+(0.5+0.5\theta-\theta q)\\&\quad\frac{(1-q\pi)\,\theta}{1-0.5\pi-(q-0.5)\,\pi\theta}.\end{aligned} \tag{25}$$

- When $s_H$ is Received and $m_L$ is Issued

$$\begin{aligned}E(\hat{\theta}|m_L,s_H)&=Pr(x_H|s_H)\cdot Pr(capable|m_L,x_H)\\&\quad+Pr(x_L|s_H)\cdot Pr(capable|m_L,x_L)\\&=(0.5-0.5\theta+\theta q)\frac{(1-q)\,\theta}{0.5-\theta q+0.5\theta}\\&\quad+(0.5+0.5\theta-\theta q)\frac{q\theta}{0.5+\theta q-0.5\theta}.\end{aligned} \tag{26}$$

- When $s_L$ is Received and $m_L$ is Issued

$$\begin{aligned}E(\hat{\theta}|m_L,s_L)&=Pr(x_H|s_L)\cdot Pr(capable|m_L,x_H)\\&\quad+Pr(x_L|s_L)\cdot Pr(capable|m_L,x_L)\\&=(0.5+0.5\theta-\theta q)\frac{(1-q)\,\theta}{0.5-\theta q+0.5\theta}\\&\quad+(0.5-0.5\theta+\theta q)\frac{q\theta}{0.5+\theta q-0.5\theta}.\end{aligned} \tag{27}$$

- When $s_L$ is Received and $m_H$ is Issued

$$\begin{aligned}E(\hat{\theta}|m_H,s_L)&=Pr(x_H|s_L)\cdot Pr(capable|m_H,x_H)\\&\quad+Pr(x_L|s_L)\cdot Pr(capable|m_H,x_L)\\&=(0.5+0.5\theta-\theta q)\\&\quad\frac{(1-\pi+q\pi)\,\theta}{1-0.5\pi+(q-0.5)\,\pi\theta}\\&\quad+(0.5-0.5\theta+\theta q)\\&\quad\frac{(1-q\pi)\,\theta}{1-0.5\pi-(q-0.5)\,\pi\theta}.\end{aligned} \tag{28}$$

It can be demonstrated that the following holds:

$$\begin{aligned}&E(\hat{\theta}|m_H,s_H)-E(\hat{\theta}|m_L,s_H)\\&=\frac{2(\theta-1)\theta^2(2q-1)^2\{\pi[\theta^2(2q-1)^2+3]-4\}}{\{\theta^2(2q-1)^2-1\}\{\pi^2\theta^2(2q-1)^2-(\pi-2)^2\}}>0.\end{aligned} \tag{29}$$

Thus, $E(\hat{\theta}|m_H,s_H)>E(\hat{\theta}|m_L,s_H)$. In other words, when considering prestige, the analyst who receives the signal $s_H$ sends $m_H$. Additionally,

$$\begin{aligned}&E(\hat{\theta}|m_L,s_L)-E(\hat{\theta}|m_H,s_L)\\&=\frac{2\pi(\theta-1)\theta^2(2q-1)^2}{\pi^2\theta^2(2q-1)^2-(\pi-2)^2}>0.\end{aligned} \tag{30}$$

Therefore, $E(\hat{\theta}|m_L,s_L)>E(\hat{\theta}|m_H,s_L)$.

That is, when considering prestige, an analyst who receives $s_L$ sends $m_L$. Thus, analysts who care about their reputation report honestly.

**Optimality of Analyst's Strategy**

From the discussion so far, an analyst who cares about the stock price sends $m_H$ regardless of whether the signal is $s_H$ or $s_L$ (equation (24)). Additionally an analyst in pursuit of fame sends $m_H$ when the signal is $s_H$ and $m_L$ when the signal is $s_L$ (equations (29) and (30)). Thus, when the signal is $s_H$, the two incentives, stock price and reputation, are not in conflict. In other words, the following holds.

$$E(P_\omega|m_H)+k\cdot E(\hat{\theta}|m_H,\ s_H) > E(P_\omega|m_L)+k\cdot E(\hat{\theta}|m_L,\ s_H). \quad (31)$$

However, if the signal is $s_L$, a dilemma arises. Specifically, if the analyst sends $m_L$ based on $s_L$, the stock price will be lower, but she will gain more prestige. Conversely, if she sends $m_H$ against $s_L$, the stock price will be higher, but her prestige will be lower. Which report to send depends on the relative degree of prestige the analyst values, that is, the parameter $k$ in equation (7). Specifically, an analyst is a good type who issues honest reports if $k$ satisfies the following:

$$E(P_\omega|m_H)+k\cdot E(\hat{\theta}|m_H,\ s_L) \leq E(P_\omega|m_L)+k\cdot E(\hat{\theta}|m_L,\ s_L). \quad (32)$$

The $k$ such that the left and right sides of equation (32) are equal is the threshold $k^*$. From this we obtain the following:

$$k^* = \frac{\pi^2\theta^2(2q-1)^2-(\pi-2)^2}{4(2-\pi)\pi(\theta-1)\theta(2q-1)}\cdot x_H. \quad (33)$$

Note that $k^*<1$. If $k^*\geq 1$, then $\pi=0$ from equation (8) and the right side of equation (33) cannot be defined. For $k^*<1$, $x_H$ must not be too large[8]. We assume this condition is satisfied.

So far, we have assumed that the analyst's strategy is to send $m_H$ in the information sets $\{s_H$, *good*$\}$, $\{s_H$, *evil*$\}$, and $\{s_L$, *evil*$\}$ and $m_L$ in $\{s_L$, *good*$\}$. Finally, given the informed trader's response and the market maker's pricing rule, we check that the analyst has no incentive to change her strategy.

- At the Information Sets $\{s_H$, *good*$\}$ and $\{s_H$, *evil*$\}$

When the analyst receives $s_H$, she sends $m_H$ to the informed trader, regardless of whether she is the good or evil type. It is clear from equation (31) that she has no incentive to send $m_L$.

- At the Information Sets $\{s_L$, *good*$\}$

If the analyst is a good type, that is, $k\geq k^*$, then equation (32) holds. Therefore, the analyst has no incentive to deviate to $m_H$.

- At the Information Sets $\{s_L$, *evil*$\}$

If the analyst is an evil type, that is, $k<k^*$, the following equation holds:

$$E(P_\omega|m_H)+k\cdot E(\hat{\theta}|m_H,\ s_L) > E(P_\omega|m_L)+k\cdot E(\hat{\theta}|m_L,\ s_L). \quad (34)$$

From this, it is clear that the analyst has no motivation to deviate to $m_L$.

Note that because the informed trader's information sets $\{m_H\}$ and $\{m_L\}$ are both on the equilibrium path, his beliefs off the equilibrium path need not be considered. ■

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[8] Specifically, $x_H < \frac{4(2-\pi)\pi(\theta-1)\theta(2q-1)}{\pi^2\theta^2(2q-1)^2-(\pi-2)^2}$ must be satisfied.

## 4. Numerical Example

In this section, we present numerical examples to demonstrate our main results and the differences from previous studies.

### Multiple Equilibria

Calculating the threshold with $\theta = 0.5$, $q = 0.6$, and $x_H = 0.01$, we get $k^* = 0.0554$ or $k^* = 0.8941$ [9]. In the case $k^* = 0.0554$, the probability that the analyst is good type from the informed trader's viewpoint (i.e., $\pi$) is 0.9446. This is a case where the analyst follows a fairly honest reporting strategy. On the other hand, in the case of $k^* = 0.8941$, $\pi$ is 0.1059. In this case, the analyst's reporting strategy appears unfaithful. Thus, in this numerical example, it is shown that there are two equilibria; one in which relatively honest reports are provided and one in which dishonest reports are frequently issued. This differs from Jackson (2005), in which at most only one equilibrium exists. Although somewhat technical, the causes of the two equilibria are as follows:

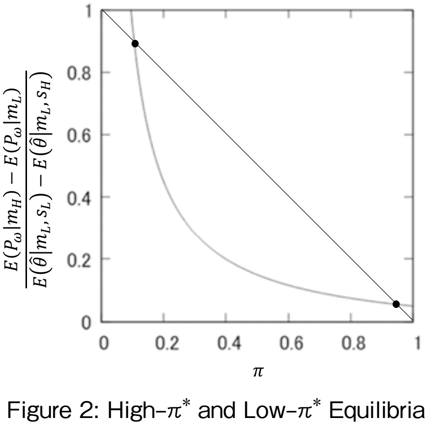


Figure 2: High-$\pi^*$ and Low-$\pi^*$ Equilibria

Based on $k = 1 - \pi$ and equation (32), finding the threshold $k^*$ is equivalent to finding $\pi$ such that

$$\frac{E(P_\omega|m_H) - E(P_\omega|m_L)}{E(\hat{\theta}|m_L, s_L) - E(\hat{\theta}|m_H, s_L)} = 1 - \pi. \quad (35)$$

The denominator on the left side of the above equation represents the prestige advantage of issuing $m_L$ (i.e., a bearish report) when $s_L$ (i.e., a dismal signal) is received. This grows with $\pi$. That is, the more honest the analyst is, the stronger the incentive to issue the report as it is. On the other hand, the numerator on the left side represents the stock price advantage of issuing $m_H$ (i.e., a bullish report). This also increases with $\pi$. Because strong confidence in the analyst's honesty makes her report persuasive, the incentive to issue a bullish report becomes strong. Thus, both the denominator and numerator increase with $\pi$. However, the numerator increases at a smaller rate than the denominator. The reason the sensitivity of the stock price effect is not so large is that market makers price for the possibility that an evil-type analyst issues $m_H$. As a result, the left-hand side of equation (35) decreases with $\pi$ and may intersect the right-hand side twice [10]. The above case is represented in Figure 2 [11].

In Jackson (2005), the denominator on the left-hand side is identical to ours, but the numerator represents the effect of stock trading commissions. Since the numerator increases at a faster rate, the value of the left-hand side increases with $\pi$. As a result, the left and right sides intersect only once at most. The fast increase in the numerator in Jackson (2005) may be due to the fact that it does not take

9 In this numerical example, the solution $k^* = -0.9990$ is also obtained, which violates assumption $k^* \in (0, 1)$. Therefore, we exclude this case.

10 Regarding the left-hand side, the first- and second-order partial derivatives with respect to $\pi$ are negative and positive. Therefore, the left-hand side exhibits a convex shape with respect to the origin.

11 Note that depending on the parameter settings, there are cases in which the left and right sides do not intersect or touch once. Here we observe a case that crosses twice.

the effect of the transaction on the stock price into account. In short, the reason for the difference between our results and Jackson's (2005) lies in the assumptions about the variability of stock price.

**Optimism in Analyst Reports**

We can identify the causes of the well-known fact that analyst reports tend to be optimistic. When there is a possibility that analysts are of the evil type, $\pi < 1$. The probability that the analyst's report is $m_H$ (i.e., $Pr(m_H)$), which is $1-0.5\pi$ from equation (20), is higher than the probability of the firm value being $x_H$. That is, analyst reports are optimistic in the sense that they frequently contain bullish outlook.

According to the numerical example, there are two equilibria, one in which relatively honest reports are provided and one in which dishonest reports are frequently issued. In the former, henceforth we describe it as a honest equilibrium, the probability that the analyst is an evil type is low (i.e., $\pi$ is large), and the probability that the analyst report is $m_H$ is 0.5277. In this case, the informed trader who receives $m_H$ updates his belief that the firm value is $x_H$ (i.e., $\lambda(m_H)$) from 0.5 to 0.5448. Thus, the analyst report has some influence. In contrast, in the latter equilibrium, we describe it as a dishonest equilibrium, the probability that the analyst is an evil type is high (i.e., $\pi$ is small), and the probability that the analyst report is $m_H$ is 0.9471. In other words, the analyst report is almost always $m_H$. In this case, the informed trader's belief (i.e., $\lambda(m_H)$) is not affected by receiving $m_H$ and is updated only up to 0.5028. In this case, the analyst report has little influence.

Thus, the news value of analyst reports differs depending on which equilibrium is realized. In the honest equilibrium, analyst reports are cautious in the sense that they have a low degree of bullish report, but they have the power to update the beliefs of informed traders. In this respect, the news value is high. In contrast, in dishonest equilibrium, analyst reports are mostly $m_H$ and thus do not have the impact of updating beliefs. In this sense, analyst reports are less newsworthy[12].

**Probability Distribution of Stock Price**

Finally, we clarify the shape of the probability distribution of stock prices in equilibrium. From the previous discussion, we know that the stock price depends on $\omega$ as in equations (16), (17), and (21). The probabilities that $P_{\omega=2}$, $P_{\omega=0}$ and $P_{\omega=-2}$ can be expressed as follows respectively.

$$Pr(m_H)\cdot Pr(u=1)=(1-0.5\pi)\cdot 0.5=0.5-0.25\pi, \tag{36}$$

$$Pr(m_H)\cdot Pr(u=-1)+Pr(m_L)\cdot Pr(u=1) =(1-0.5\pi)\cdot 0.5+0.5\pi\cdot 0.5=0.5, \tag{37}$$

and

$$Pr(m_L)\cdot Pr(u=-1)=0.5\pi\cdot 0.5=0.25\pi. \tag{38}$$

The probability distribution of stock prices is illustrated in Figure 3. The horizontal and vertical axis represent the stock price level and the probability of occurrence, respectively; open and stippled bars represent honest equilibrium and dishonest equilibrium cases, respectively. Both cases have a common probability of 0.5 that the stock price is $P_{\omega=0}$. However, while the probability distribution for the honest equilibrium case has a nearly symmetric shape, it is highly skewed for the dishonest equilibrium case. Specifically, the probability of having $P_{\omega=-2}$ is high at 0.474, while that of having $P_{\omega=-2}$ is extremely low.

The reason the probability distribution of the dishonest equilibrium case has such a skewed shape can be explained as follows. Since most analyst

[12] It should be noted that this analysis does not provide predictions about which of the two equilibria will hold.

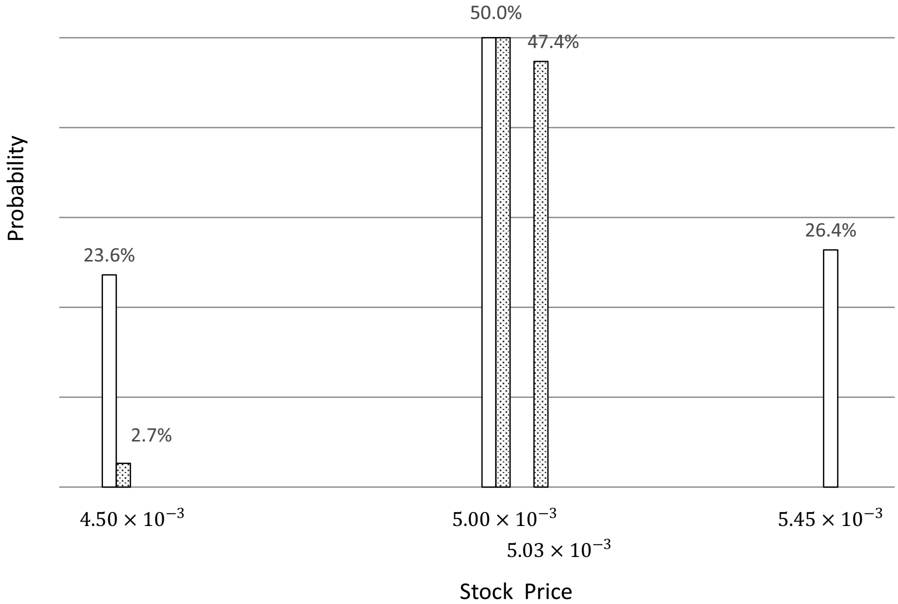


Figure 3: Probability Distribution of Stock Price

reports are $m_H$, the probability that an informed trader submits a buy order is also close to 1. As a result, the probability of $P_{\omega=-2}$ is significantly low, and that of $P_{\omega=2}$ approaches 0.5. On the other hand, a market maker expects analyst reports to be very optimistic, so the value of $P_{\omega=2}$ is discounted. In fact, the value of $P_{\omega=2}$ is $5.03 \times 10^{-3}$, almost the same as $P_{\omega=0}$. For these reasons, the probability distribution of dishonest equilibrium case is skewed. This distortion becomes more severe as an analyst becomes more dishonest. Then, in the limit of $\pi \to 0$, the stock price has a probability of 1 to be $5.00 \times 10^{-3}$. At this point, the analyst report does not provide any information. In contrast, the distortion is corrected as the analyst's honesty increases. In the limit of $\pi \to 1$, the distribution is symmetric.

## 5. Conclusion

We demonstrate that stock price considerations can be a source of optimism bias in analyst reports. We derive an equilibrium consisting of the analyst's reporting strategy and the informed trader's trading strategy. The analysis reveals that an equilibrium exists in which the analyst adopts an optimistic reporting strategy to stimulate buy orders and the informed trader adopts trading strategy that follows the analyst's report. In addition, two equilibria exist, one in which relatively honest reports are provided and one in which dishonest reports are frequently issued. We also found that analyst honesty has an effect on the optimism and news value of analyst reports, as well as on the probability distribution of stock prices.

Depending on which of these equilibria is realized, the economic significance of analyst reports differs considerably. In particular, the different news values of reports are expected to affect analysts' economic welfare and investment outcomes. This study does not consider these points. Therefore, these are issues that remain for the future.

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